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Experimental assessment of the productivity improvement when using U-shaped production cells with variable takt time

Patrick Pujo, Ilham El Khabous, Fouzia Ounnar

Aix-Marseille University

CRET-LOG (Centre de Recherche sur le Transport et la LOGistique)

Avenue Escadrille Normandie Niémen - 13397 Marseille cedex 20 – France

patrick.pujo@univ-amu.fr,

ilham.elkhabous@gmail.com

fouzia.ounnar@univ-amu.fr

Experimental assessment of the productivity improvement when using U-shaped production cells with variable takt time

Purpose – The aims of this research is to discuss the benefits of U layout for production cell operating in variable takt time. Different experiments were conducted using benchmarks, in order to highlight the performance gap between a straight cell and a U-cell.

Design/methodology/approach – The implementation of the production cell, in U-shaped or in straight line, is optimized through linear programming based on the number of operators. The two corresponding programs, in Mosel language, use the same approach, in order to not introduce defects in the comparison of results. The study used our own datasets and well known academic benchmarks.

Findings – A comparison between the obtained takt times, with equivalent operating conditions, in U-cell and straight cell was conducted. A significant increase of the production rate was observed. This increase has often exceeded 10 per cent to reach 32 per cent. All the experiments show that, with the same number of operators, a cell in a U layout is always at least as efficient, in terms of reachable production rates, than an equivalent cell in a linear layout. 96 per cent give an improvement of production rate. Moreover, the dispersion of the U-cell results is weaker, which suggests that the U-layout gives, in more robust manner, better performances.

Research limitations/implications – The results were obtained through a study of various academic benchmarks. The results must be validated on industrial situations.

Practical implications – This paper will be very useful for researchers and practitioners in order to understand lean implementations and its derived benefits. This paper will allow them evaluating and analysing the expected benefits of the implementation of the production cell in U-shaped (operating in variable takt time).

Originality/value – U-Cells constitute an appropriate solution for a layout of any kind of production cells with variable structure. When facing a significant variation in the demand, the response consists in modulating the number of operators assigned to the cell. This study addresses jointly the problem of U-cells layout and the operation in variable takt time.

Keywords: U-shaped production cell, straight production line, production variable throughput, linear programming for optimization, assembly line balancing.

1. Introduction

Recent developments towards lean management (Holweg, 2007) reflect an increasing trend by companies to use this approach. This is often found in the literature in the form of testimonials: the effectiveness of lean management methods and tools are typically expressed through case studies (Detty and Yingling, 2000), (Motwani, 2003), (Modarress *et al.*, 2005), (Kumar *et al.*, 2006). For example, tools such as value stream mapping (VSM) have been the subject of evaluation 'before & after' comparison, used alone (Singh *et al.*, 2010) or in combination with other techniques in the case of production lines

(Alvarez *et al*., 2009). Based on value stream mapping and milkrun, using lean metrics, such as dock-to-dock time and lean rate, these works analyse the internal materials flow and its improvement.

Other tools, such as U-shaped cells producing or takt time variable, have not been subjected to comparison 'before & after'.

U-shaped production cells (figure1(b)) emerged in the Western industrialized world with the introduction of the Just in Time philosophy (Monden, 1983) and are currently associated with the Lean Production approach (Shah and Ward, 2003).

The majority of the production lines not identified as falling within one of these two approaches are organized linearly (figure 1(a)). The optimization of their performance is obtained using load balancing techniques which are dealt with in numerous studies (Becker and Scholl, 2006).

Please insert here Figure 1. Straight production cell (a), U-shaped production cell (b)

U-Cells offer many advantages over traditional linear arrangements, in terms of flexibility, standardization, decrease in work in progress and the maintaining of high quality products. They also provide improvements in productivity.

In the case of high variations in production volumes, it is easy to adjust the number of operators to adapt the production rate to the instantaneous demand (takt time). Through variable takt time, the production lines enable new strategies, so that lean product realisation can become a reality for variable productions (Weston and Cui, 2008). This is common in U-Cells, but can also be achieved in straight production lines. It becomes then interesting to compare both layout approaches and evaluate their respective performances through experiments using benchmarks.

After defining the basic principles of the operation of a variable rate production line, a few fundamentals regarding U-Cells are recalled. Then, the issue of the layout optimization for such systems is addressed and linear programming models are developed aiming to obtain the best possible balancing of the operator tasks.

These models are then implemented into a solver and tested on academic benchmarks which are set to be the most impartial and the least specific possible.

Finally, the results of these experiments are analysed and discussed.

2. Choice of a layout architecture for a production cell

2.1. Cellular production: some reminders

Production lines were deployed at the beginning of the 20th century, after Henry Ford installed the first assembly line in 1913 (Sward, 1989). After being used for a century in mass manufacturing, this concept is still evolving today (Hirst and Zeitlin, 1991), being revisited (Bartholdi and Eisenstein, 1996). This history, combined with a pragmatic quest for efficiency (Magee, 2007), led the Japanese industry to develop the idea of production cells: a production cell is a small production unit which specializes in the manufacturing of a family of products or parts.

This type of organization should offer greater usage flexibility, better performances in terms of quality, cost and time, as well as a fluidization of flows combined with reduction in the volume of work in progress and greater responsiveness. In fact, a cell is a small production line whose conditions of use can vary according to needs: change of rate, of products, of the number of machines, etc. The line architecture is related either to the grouping together of all the processes related to the fabrication of a family of products, or to the grouping resulting from a frequent series of operations common to the manufacturing of different types of products.

When this approach is generalized to the whole production system, we can refer to it as cellular manufacturing (Irani, 1999): the variety of products to be manufactured, their estimated volumes and the fluctuations in demand will determine the set of cells which will need to be formed and designed. Numerous studies about cellular production focus on the characterization of this system using different approaches such as group technology (Kulak *et al.*, 2005), integer linear programming (Arifar and Ismail, 2009), metaheuristics of genetic algorithm type (Yin and Yasuda, 2006) or ant colony optimization algorithms (Sabuncuoglu *et al*,. 2009).

Another emerging problem is related to the dynamics of the control of this set of cells (Wenming and Deuse, 2009). For instance, if we consider a workshop where many neighbouring cells respond to varying and desynchronized customer loads, it is possible to adjust the number of operators per cell and transfer operators within cells (communicating vessels) so that the workshop is fully controlled and maximum productivity is achieved, preserving responsiveness and dynamics whatever the variation of customer demands and providing a high potential increase in production rate. For example, a cell functioning with n operators (nominal operation) can be adjusted to $n-1$ or $n+1$, but it is possible to make it function even faster, with up to $n+k$ operators, the additional operators being taken from neighbouring cells or even make it function very slowly (with only one operator) or stop it temporarily.

In any case, the search for the optimal cell productivity requires the balancing of the production loads: the key is to set up the cell so it can produce at the lowest cost possible while complying with the technical constraints.

2.2. Balancing of a production cell

This issue was first raised in automotive assembly lines, and is now referred to in the scientific literature as SALBP (Simple Assembly Line Balancing Problem) (Baybars, 1986). A typology of the resolution methods for this type of problems was suggested by (Boysen *et al.*, 2007). Generally speaking, these methods consist in splitting the task as effectively as possible between the operators so that the workload is equally distributed and the production rate is optimal.

The general features of this problem are the following:

- a single operation cannot be shared between different operators;
- the operations are partially ordered;
- all operations must be executed;
- durations of operations must not depend either on the operator or on previous operations;
- an operator can carry out any operation;
- any operation can be executed by any operator;
- the products pass one by one in front of operators in a given order;
- the operators themselves are positioned in a given order;
- one single product is produced per line.

Operations have to be split between operators so that waiting times are minimal. The waiting time of each operator is determined by the difference between the cycle time of the line (the load of the busiest operator) and the actual load (the sum of all the times of operation assumed by the operator). The solution in which the operator waiting time is the shortest provides the lowest cost.

Two cases are possible. First, if the operators are assigned to more or less versatile workstations (for example, in the case of an assembly workshop), the balancing will consist in assigning them a maximum number of operations. Then, if the operators have to move between different workstations, the balancing will consist in assigning them various consecutive workstations, with a displacement cycle between these workstations.

Even though the mathematical formulation of these two cases is equivalent, our interest will be mainly focused on the second case. The displacement of operators between different workstations is called Chaku-Chaku (Wenming and Deuse, 2009).

Let us note that the so called "caravan system" (Spengler *et al.*, 2005), which consists in moving k operators (out of phase by 2π/k) along the entire cell (figure 2(a)), with an identical Chaku-Chaku, is not taken into account. We will focus on the balancing of the line between different Chaku-Chaku (one operator corresponding to one Chaku-Chaku), involving a manual transfer of parts between neighbouring Chaku-Chaku operators (figure 2(b)).

Please insert here Figure 2. Chaku-Chaku in ‘caravan’ system (a), Chaku-Chaku with manual transfer (b)

It is also important to detail the nature of tasks T_i which refer to the operations carried out by operators O_k on the different workstations P_i. We should distinguish between durations Do_i of tasks T_i and displacement times between workstations Dd_{xi}. The latter are usually small compared to the cumulative durations of tasks performed by each workstation and they can be either neglected or integrated to the corresponding durations: Do_i includes Dd_{xi}. Besides the displacement time, the duration includes product loading/unloading tasks on the workstation, possible product control tasks, and either the execution of this task - in the case of a purely manual task – or the initiation of the execution of the task followed by a short period of monitoring to visually check whether the task has started correctly - in the case of an automated workstation.

This refers to the apparent time component D_i of the operation on workstation P_i. For automated tasks, the continuation and completion of the task progress in masked time, on a period called Dm_i. Note that $Do_i = D_i + Dm_i$. To each workstation P_i corresponds a chronogram of all the tasks of the operation (Monden, 1983), called 'Standard Operation Chart' and all the chronograms show the cyclic activity of operator O_k in a Gantt diagram, called 'Standard Operation Combination Chart',

If T_k is the cycle time of operator k, the following relationships must be fulfilled:

- For a given operator k, to which n_k operations i have been assigned:

$$\sum_{j=1}^{n_k} D_{i_j} < T_k \tag{1}$$

- For each given operation i:

$$D_i + Dm_i < T_k \tag{2}$$

- If T is the cycle time of the cell and n_p the number of operators, then:

$$\forall\ k \in [1, n_p],\ \max(T_k) = T \tag{3}$$

These two overall variables strongly influence the resolution method, depending on whether they are fixed or to be minimized. SALBP can be expressed in 4 ways (Baybars, 1986), (Becker and Scholl, 2006):

Please insert here Figure 3. Classification of line balancing problems

SALBP-F is the feasibility study of a configuration, and not a search for optimization. SALBP-1 studies the minimization of the number of operators while respecting a given production rate. SALBP-2 aims at minimizing the cycle time so as to achieve a maximum production rate through a fixed number of operators. SALBP-E combines those two options and maximizes the efficiency of the production line

These SALBP problems deal with the balancing of straight cells. The layout of U-shaped production cells is the subject of specific studies collectively known as SULBP (Simple U-Line Balancing Problem) (Aase *et al.*, 2004) or UALBP (U-Assembly Line Balancing Problem) (Scholl and Klein, 1999b).

2.3. Balancing of a U-Cell

For U-shaped cells, the classification of the resolution methods is similar (Kriengkorakot and Pianthong, 2007). (Boysen *et al.*, 2007) identifies a number of jobs, denoted by [|u|].

Regarding SULBP-1, (Miltenburg, 1998) proposed first a dynamic programming approach, then an integer linear programming approach (Miltenburg, 2001b). More recently, (Sabuncuoglu *et al.*, 2009) suggested a resolution using an ant colony metaheuristic.

Other studies relate to SULBP-E: for example, (Chiang and Urban, 2006) used a heuristic procedure; (Kara *et al.*, 2009) applied a multi-objective fuzzy integer programming method.

Finally, a number of studies address the Mixed Model U-Line Balancing and Scheduling Problem (MMULBS) regarding the production of a variety of products in a U-Cell, simultaneously considering balancing and scheduling (Miltenburg, 2002), (Kim *et al.*, 2006), (Kazemi *et al.*, 2011).

2.4. Performance comparison between the balancing of a straight cell and a U-shaped cell

Numerous studies were undertaken simultaneously on both approaches (Ağpak and Gökçen, 2007, Kara *et al.*, 2009), but focusing on the way the production cells were organized and thus on the cell architecture generation algorithm, without comparing their performances with equivalent datasets.

As our study is performed from a perspective of variable takt time, we can compare both types of layout by changing the number of operators and comparing the respective production rates obtained. For that, we should move towards SALBP-2 and SULBP-2 approaches.

3. Modelling of the SxLBP-2 balancing problem for a production cell

In this section, the implemented linear programming models are described. The two models have to be homogeneous in terms of structure and design in order to avoid any methodological biases. As there is comparison, we chose to consider one of the layouts as a reference point, in terms of modelling, implementation and results analysis. Based upon the literature review, we chose to start from an existing SALBP-2 approach and extend it to a U-Cell.

3.1. Modelling of the SALBP-2 problem

The SALBP-2 model described here results from (Guéret *et al.*, 2002). This model is relatively simple, gives excellent results and its implementation is already using Xpress solver. The following notations are considered:

- n_t: number of operations,
- D_i: duration of operation i ($i \in [1 \ldots n_t]$),
- n_p: number of operators,
- n_a: number of arcs (sequence of operations). An arc (u, v) links operation u to operation v if u is the immediate predecessor of v.

In terms of modelling, x_{ik} binary variables will be used to place the operations:

- $x_{ik} = 1$ if and only if operation i, is assigned to operator k (see *(4)* & *(8)*).

For all valid assignments, in accordance with *(4)* and *(8)*, it is also necessary to respect precedence constraints: for any arc (u, v), the rank of operation u must be lower than or equal to that of operation v (see *(5)*).

Finally, a real variable $T \geq 0$ is used (see *(9)*) for the cycle time of the cell, which according to *(6)* is equal to the cycle time of the busiest operator. The objective function consists in minimizing T (see *(7)*).

The formulation of the SALBP-2 problem is as follow:

$$\forall i=1 \ldots n_t: \sum_{k=1}^{n_p} x_{ik} = 1 \tag{4}$$

$$\forall (u,v) \in [1 \ldots n_t]: \sum_{k=1}^{n_p} k.\ x_{uk} \leq \sum_{k=1}^{n_p} k.x_{vk} \tag{5}$$

$$\forall k=1 \ldots n_p: \sum_{i=1}^{n_t} D_i.x_{ik} \leq \mathrm{T} \tag{6}$$

$$Min\ z = T \tag{7}$$

$$\forall i=1 \ldots n_t,\ \forall k=1 \ldots n_p: x_{ik} \in \{0,1\} \tag{8}$$

$$T \geq 0 \tag{9}$$

Under these conditions, the first operator handles the cell inputs while operator n_p manages the outputs.

3.2. Extending the modelling of SALBP-2 to SULBP-2

The U shaped-cell layout involves treating simultaneously incoming and outgoing branches of the U-cell. Let j be the branch index, with $j = 1$ for the incoming branch and $j = 2$ for the outgoing branch.

Modelling the SULBP-2 problem thus requires the binary variables to adjust to this notion of treating simultaneously the two branches to position the operations.

- x_{ijk} = 1 if and only if operation i is assigned to operator k on branch j (see *(10)* & *(15)*).

For all valid assignment, in accordance with *(10)* & *(15)*, it is also necessary to respect precedence constraints: for any arc (u, v), the rank of operation u must be lower than or equal to that of operation v. However, the calculation of this rank can differ according to the branch. In branch 1, we are back to the previous case (see *(11)*). In branch 2, the precedence must be reversed (see *(12)*).

The principle of minimization of the busiest operator cycle time remains unchanged (see *(13)*), and takes into consideration the two branches, *(14)* & *(16)*).

The formulation of the SULBP-2 problem is as follows:

$$\forall i=1 \ldots n_t: \sum_{j=1}^{2} \sum_{k=1}^{n_p} x_{ikj} = 1 \tag{10}$$

$$\forall (u,v) \in [1 \ldots n_t]: \sum_{k=1}^{n_p} k . x_{uk1} \leq \sum_{k=1}^{n_p} k . x_{vk1} \tag{11}$$

$$\forall (u,v) \in [1 \ldots n_t]: \sum_{k=1}^{n_p} k . x_{vk2} \leq \sum_{k=1}^{n_p} k . x_{uk2} \tag{12}$$

$$\forall k = 1 \ldots n_p: \sum_{j=1}^{2} \sum_{i=1}^{n_t} D_i . x_{ikj} \leq \mathrm{T} \tag{13}$$

$$Min\ z = T \tag{14}$$

$$\forall i = 1 \ldots n_t, \forall k = 1 \ldots n_p, \forall j = 1, 2: x_{ikj} \in \{0,1\} \tag{15}$$

$$T \geq 0 \tag{16}$$

In this formulation of the SULBP-2 problem, operator n_p manages the inputs and outputs of the cell. Operator 1 is situated in the heart of the cell.

3.3. Implementation of the models

The implementation of these models is performed using Mosel language and Xpress solver, available under FICO optimization environment (www.fico.com).

Thereafter, only the programming of the SULBP-2 problem (figure 4) will be described, that of the SALBP-2 problem having already been described in (Guéret *et al.*, 2002).

The binary assignment variable x is represented by a 3D table (n_t, n_p, 2), the third dimension being used to separately treat the two branches of the U.

There are n_t constraints '*OneOperationPerOperator(i)*' which express the fact that an operation i cannot be shared between several operators.

There are n_p constraints '*Cycle (m)*' which express the fact that the sum of operations assigned to an operator m is lower than or equal to the cycle time of the U-Cell.

Finally, the constraints '*Prec_branch_input(a)*' and '*Prec_branch_output(a)*' allow the precedence constraints between operations to be taken into consideration for each sequence of operations and whatever the assignment of an operation on one or the other of the U-Cell branches.

The number of operators is defined by the declared variation range of the variable '*OPERATORS*'.

Please insert here Figure 4. SULBP-2 problem programmed in Mosel language

4. Experimentations with simple linear precedence graph

4.1. Establishment of the dataset

The dataset presented in Table 1 is an academic benchmark which was precisely elaborated for the specific purpose of controlling the variability of a number of parameters, in order to establish a performance comparison between U-shaped and straight cells.

Therefore, all the 34 tested cases were built by altering only the duration of the tasks, according to a certain number of invariants: number of operations (17 workstations), total task duration (303 Time Units), duration of the shortest task (5 TU), duration of the longest task (60 TU) and purely linear flow: the output flow of operation u goes directly into operation $u+1$, which induces a linear precedence graph. The above mentioned models would naturally support other values for these invariants: this is to test different configurations of task durations with a variable number of operators, on linear or U type layouts.

Please insert here Table 1. Description of the dataset: the 34 studied cases.

4.2. Experimentation results

Thereafter, only the result of case n°1 (figure 5) will be detailed. For x operators, the distribution of the 17 operations is given, for SALBP-2 (Lx) and SULBP-2 (Ux), as well as the cycle durations for each operator and the takt time of the associated cell. Each line also indicates the Chaku-Chaku of the operator concerned.

Please insert here Figure 5. Case n°1 results

The 34 studied cases are summarized in Table 2. For each Lx & Ux configuration, the cycle durations and the improvement percentage are given.

We observe that for one single operator, the results are the same in a straight or U-shaped cell. This is a trivial result. This experiment is thus not significant.

For 2 or 3 operators, the results are equivalent in rare cases: the advantage generally is in favour of U-Cells.

Please insert here Table 2. Summary of the experimentation results (in bold: improvements greater than 10%)

Similarly, for a higher number of operators, optimization is no longer efficient.

This can be easily explained since the cycle duration is bounded by the bottleneck due to the longest task assigned to the cell: Lx and Ux results converge, either for 7 operators, or starting from 6 operators. These cases are not significant. Only case 32 achieves a slight performance advantage with 7 operators, advantage which is lost with 8 operators. Between these two extreme cases (123 significant experiments), the results are always in favour of a U-shaped layout, very often with improvements in cycle time exceeding 10%. Based on these significant experiments, 117 provide an improvement of the performance (95%). This result is substantial.

4.3. Analysis of experimentation results

Table 3 shows a summary of the previous results. An average 8% improvement of the maximum possible production rate is obtained, with peaks up to 25%. Figure 6 displays the scattering of the results.

Please insert here Table 3. Average and standard deviation of cycle durations of the various configurations, for Lx & Ux ($x \in [2, 6]$)

Please insert here Figure 6. Distribution of the results, for Lx & Ux ($x \in [2, 6]$)

We note that the scattering of the results is much greater for a linear layout than for a U-shaped cell, which demonstrates the robustness of the U-shaped layout approach in the case of a linear precedence graph.

5. Experimentations with a complex precedence graph

In many manufacturing systems, flows are not always fully ordered: tasks are partially organized by precedence relations. For instance, if there are jumps between operations, or if operations can be performed in parallel or independently, these anteriority constraints between tasks can be modelled by a precedence graph, in which tasks are the nodes and precedence constraints the arcs. In the previous models, the *ANT* table (Figure 4) makes it possible to present each precedence relationship as an elementary flow between two operations.

The whole is then a network of precedence constraints.

We will examine the constraints induced by these considerations when operating with a variable takt time.

5.1. Problem of implementation with a complex precedence graph

The optimization models presented in the 3rd section for the resolution of SALBP-2 and SULBP-2 provide excellent solutions regardless of the precedence graph. Nevertheless, they cause a major problem in the case of an operation with takt time varying with regular frequency: for instance if we wish to adapt on a daily basis the operation to the customer's demand. Indeed, when the number of operators is changed the order in which the tasks are arranged on the production line can be changed. However, it is unconceivable to move resources and workstations at each takt time variation. This of course does not occur when the precedence graph is strictly linear, as seen in paragraph 4.

To overcome this difficulty, we must linearize the complex precedence graph in order to identify the most appropriate linear precedence graph that would be able to support the different chaku-chaku configurations. It is thus necessary to identify the linear precedence graph without any back-tracking, regardless of the chaku-chaku configuration applied. So as to realize this linearization while respecting this goal, we propose a two-step process. First of all, a breakdown into chronological levels uses the 'Activity on node' approach applied in project management. Then, the total moment of From-to chart is minimized at each level.

5.2. Methodology for linearizing a complex precedence graph

The complex 'Activity on node' diagram is first broken down into levels (Elmaghraby, 1977). The principle is the one used in 'precedence diagramming method' approaches (Wiest, 1981), (Tavares, 1990). The first level contains activities with no antecedents. These activities are removed from the activity list, and we start the same process again with the remaining activities to get to the next level (Crandall, 1973). This allows us to structure the precedence graph with a global order.

Then, an order has to be found within activities on the same level. To do so, the notion of total moment Z of From-to chart is applied (Esgin *et al.*, 2010):

$$Min\, Z = \sum_{i=1}^{N} \sum_{j=1}^{N} f_{i,j}\ |j - i| \tag{17}$$

where $f_{i,j} = 1$ if a precedence relation exists between the 2 activities, and $f_{i,j} = 0$ otherwise.

From–to chart is the relationship flow matrix. It gives the dominance of the flows between resources and indicates the closeness of resources in a facility layout design (Mahdavi *et al.*, 2008). From-to chart could be classified as loose and tight (Moore, 1980), depending on whether it contains respectively a few and a large number of strong precedence constraints. In our case, the From-to chart contains many entries with zero values: this corresponds to the case of flow line layout (Hassan, 1994).

To minimize this moment, we proceed to activity permutations at the same level. This allows us to find a local order which remains compatible with the previously established global order.

The linear sequence which results from this treatment can be used as a reference sequence to apply SALBP-2 and SULPB-2. Other sophisticated mathematical techniques may also be applied, but what interests us here is the possibility to compare SALBP-2 and SULBP-2 on equal and logical datasets. Thanks to this approach, we can consider that all cases with complex precedence graphs can be transformed in cases with simple linear precedence graphs. We can then use the approach mentioned in sections 3 & 4.

6. Experimentations with well-known benchmarks

More than 250 benchmark instances are presented in (Scholl and Klein, 1999a). A complete description of the datasets can be downloaded from the web at: http://alb.mansci.de/files/uploads/UALBP-1 data sets.zip

It seemed interesting to us to test some of these instances to verify our results. We considered 12 instances containing complex precedence graphs: 4 small-sized and 8 large-sized datasets.

6.1. Small-sized benchmarks

The first 4 instances can be considered small since the number of tasks as well as the number of required operators is around 10. A description of these instances is given in Table 4.

Please insert here Table 4. Description of small-sized instances

Table 4 gives an idea on the distribution of task durations and indicates the linear sequence equivalent to the initial complex precedence graph.

The results are shown in Table 5.

Please insert here Table 5. Results for small-sized instances

The performance is improved in 14 significant experiments among 16 (87.5%).

6.2. Large-sized benchmarks

The same principle was applied to large-sized datasets. Table 6 describes the datasets.

Please insert here Table 6. Description of large-sized datasets.

The results are given in Table 7

Please insert here Table 7. Results for large-sized datasets.

90 significant experiments among 91 allow improving the performance (99%).

6.3. Analysis of the results

This series of tests confirms the previous observations while leading to new observations.

For small-sized instances, the results confirm the performances achieved, similar to those presented in section 4. We will observe 32% performance improvement for the Jackson instance with 3 operators.

For large-sized instances, improvements are small for low rate productions (number of operators equal to two or three). On the other hand, for higher production rates (lower takt time), improvements are substantial, easily exceeding 10%. We also note that the maximum production rate is more quickly reached with a U layout, since the maximum rate limit, defined by the longest task duration, is reached with fewer operators: out of the 8 cases tested, we note that for [‘Heskia’], the highest rate is reached for a U layout with 11 operators while 12 would be needed in a straight line, for [‘Gunther’, Lutzl’ & ‘Mitchell’], the maximum rate is reached with 2 fewer operators, and for 3 others [‘Buxey’, ‘Kilbrid’ & ‘Sauvey30’], with 3 fewer operators, i.e. almost 1/5 of the payroll. This is a meaningful result.

7. Conclusion

All the experiments presented here show unequivocally that, with the same number of operators, a cell in a U layout is always at least as efficient, in terms of reachable production rates, than an equivalent cell in a linear layout. In summary, in order to produce more with similar loads, U-Cell layouts are needed.

This was shown through our experiments and also with academic benchmarks. Based on all tested cases (230 significant experiments), 96% of cases give an improvement of performance. This remains of course to be validated on industrial situations.

The productivity gains observed are substantial. Furthermore, the performance robustness obtained makes it possible to validate this approach in the case of a manufacturing production subject to wide variations. Indeed, regardless of the rate requested by the downstream production, the *ad hoc* configuration of the U layout will be the most suitable for obtaining an optimal performance. This shows the great interest of the U-shaped cell layout approach.

We conclude that U cells constitute a solution for implementing any relevant and effective production system with variable structure. It is an essential tool for a lean manufacturing approach.

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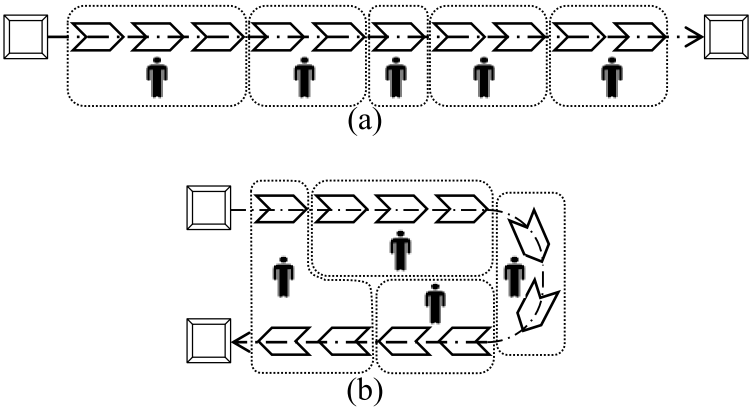

Figure 1. Straight production cell (a), U-shaped production cell (b)

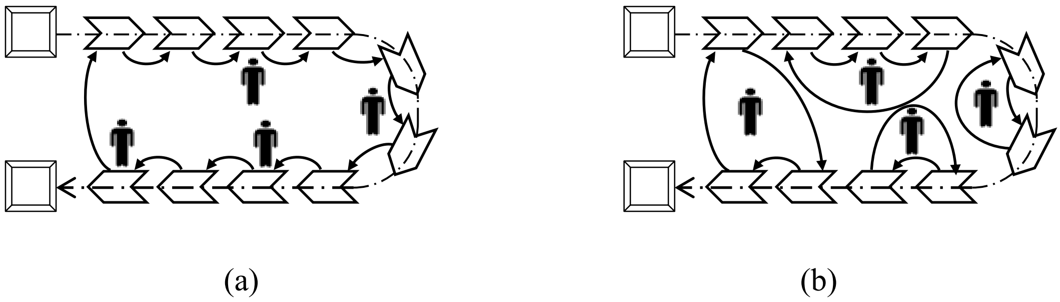

Figure 2. Chaku-Chaku in ‘caravan’ system (a), Chaku-Chaku with manual transfer (b).

	Cycle time (rate)	
Number of workstations	Fixed	To minimize
Fixed	SALBP-F	SALBP-2
To minimize	SALBP-1	SALBP-E

Figure 3. Classification of line balancing problems

```
! SULBP-2

model "U_cell balancing"
 uses "mmxprs"

 declarations
  OPERATORS=1..5                                ! All operators : here, np=5
  OPERATIONS=1..17                               ! All operations
  POS=1..2                                       ! POS=1 : input cell branch,
                                                 ! POS=2 : output cell branch

  DUR: array(OPERATIONS) of integer              ! Task duration of operation
  ANT: array(RA:range, 1..2) of integer          ! Precedence relations between operations

  x: array(OPERATIONS,OPERATORS,POS) of mpvar   ! 1 if the operator performs the operation
                                                 ! 0 else
  cycle: mpvar                                   ! Duration of the production cycle
 end-declarations

 initializations from 'U_cell balancing data1.dat'
  DUR ANT
 end-initializations

! an operation performed by an operator
 forall(i in OPERATIONS) AnOperationByOperator(i):= sum(m in OPERATORS, j in POS) x(i,m,j) = 1

! Sequence of operations
 forall(a in RA)
  do
     Prec_branch_input(a):= sum(m in OPERATORS) m*x(ANT(a,1),m,1) <=
                                  sum(m in OPERATORS) m*x(ANT(a,2),m,1)

     Prec_branch_output(a):= sum(m in OPERATORS) m*x(ANT(a,2),m,2) <=
                                  sum(m in OPERATORS) m*x(ANT(a,1),m,2)
  end-do

! Cycle time
 forall(m in OPERATORS) Cycle(m):= sum(j in POS) sum(i in OPERATIONS) DUR(i)*x(i,m,j) <= cycle

 forall(i in OPERATIONS, m in OPERATORS, j in POS) x(i,m,j) is_binary

! Minimize the production cycle time
 minimize(cycle)
```

Figure 4. SULBP-2 problem programmed in Mosel language

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	∑			
case n°1	5	23	12	10	15	5	30	8	12	10	20	32	9	24	13	60	15	303			
1 operator																					
Straight production cell	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	303	1 operator	L1	303
U-shaped production cell	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	303	1 operator	U1	303
2 operators																					
Straight production cell	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	150	1th operator	L2	153
	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	153	2nd operator		
U-shaped production cell	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	151	1th operator	U2	152
	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	152	2nd operator		
3 operators																					
Straight production cell	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	108	1th operator	L3	108
	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	107	2nd operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	88	3° operator		
U-shaped production cell	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	98	1th operator	U3	103
	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	102	2nd operator		
	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	103	3° operator		
4 operators																					
Straight production cell	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	70	1th operator	L4	80
	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	80	2nd operator		
	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	78	3° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	75	4° operator		
U-shaped production cell	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	74	1th operator	U4	77
	0	0	0	1	1	1	1	1	0	0	0	0	1	0	0	0	0	77	2nd operator		
	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	77	3° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	75	4° operator		
5 operators																					
Straight production cell	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	65	1th operator	L5	75
	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	43	2nd operator		
	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	74	3° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	46	4° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	75	5° operator		
U-shaped production cell	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	62	1th operator	U5	64
	0	0	0	0	0	1	1	1	1	0	0	0	1	0	0	0	0	64	2nd operator		
	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	62	3° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	60	4° operator		
	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	55	5° operator		
6 operators																					
Straight production cell	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	65	1th operator	L6	65
	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	55	2nd operator		
	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	62	3° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	46	4° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	60	5° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	15	6° operator		
U-shaped production cell	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	60	1th operator	U6	60
	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	40	2nd operator		
	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	51	3° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	37	4° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	60	5° operator		
	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	55	6° operator		
7 operators																					
Straight production cell	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	50	1th operator	L7	60
	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	58	2nd operator		
	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	42	3° operator		
	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	41	4° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	37	5° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	60	6° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	15	7° operator		
U-shaped production cell	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	50	1th operator	U7	60
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	32	2nd operator		
	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	44	3° operator		
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	15	4° operator		
	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	47	5° operator		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	60	6° operator		
	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	55	7° operator		

Figure 5. Case n°1 results

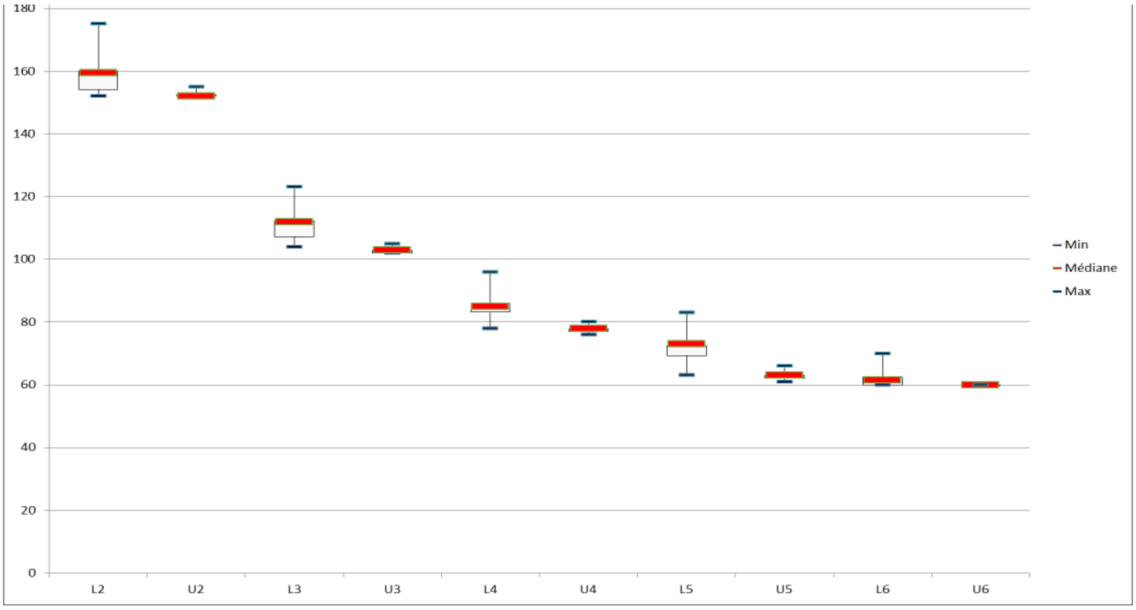

Figure 6. Distribution of results, for Lx & Ux ($x \in [2, 6]$)

Table list

Case n°	*P1*	*P2*	*P3*	*P4*	*P5*	*P6*	*P7*	*P8*	*P9*	*P10*	*P11*	*P12*	*P13*	*P14*	*P15*	*P16*	*P17*
1	5	23	12	10	15	5	30	8	12	10	20	32	9	24	13	60	15
2	1	9	17	60	5	14	22	50	11	29	10	20	7	10	6	9	14
3	1	60	15	5	13	22	30	40	4	16	20	20	14	7	10	8	9
4	5	8	5	11	10	12	12	15	13	17	14	19	20	24	23	35	60
5	5	10	20	18	25	20	52	6	60	11	8	22	11	9	5	10	11
6	5	15	20	60	35	10	8	9	50	12	41	5	7	10	6	5	5
7	1	5	12	34	24	18	14	42	11	60	16	8	10	15	6	8	9
8	1	60	22	15	9	6	56	10	16	6	7	11	13	8	20	5	21
9	1	21	25	5	22	27	14	24	60	11	9	13	5	9	13	20	8
10	5	9	16	15	13	60	16	5	50	34	10	15	6	9	14	8	18
11	1	5	60	7	16	24	11	31	12	9	11	28	19	5	39	7	9
12	1	21	22	60	5	19	27	17	23	12	16	8	10	18	11	7	13
13	5	60	51	18	39	6	12	18	11	21	6	6	7	14	7	14	8
14	6	8	14	36	13	18	7	25	5	5	17	16	12	30	6	10	21
15	2	11	8	5	60	14	30	6	12	8	25	15	36	27	7	9	10
16	1	8	15	22	18	5	60	25	29	6	18	26	9	13	19	7	13
17	5	20	20	5	5	30	60	20	8	5	30	40	10	5	20	10	10
18	1	35	5	60	23	8	20	11	8	17	15	24	6	16	14	22	9
19	6	5	40	30	20	10	5	7	10	22	20	39	8	5	7	10	5
20	6	5	25	10	15	7	30	5	20	6	23	10	9	49	60	7	16
21	5	23	14	25	11	9	7	30	13	29	8	30	9	5	60	17	8
22	3	37	22	9	34	11	12	10	7	8	60	5	9	20	6	8	15
23	5	12	29	35	8	7	10	30	14	9	8	60	10	7	15	23	21
24	8	17	60	6	12	15	9	17	20	22	26	5	21	24	13	22	6
25	1	6	15	17	10	60	30	24	5	20	10	16	9	12	10	18	29
26	9	6	12	19	5	7	31	24	17	60	11	15	10	18	24	7	28
27	1	25	5	32	12	15	11	60	12	10	11	35	11	5	22	15	7
28	1	15	5	30	23	8	10	15	12	14	16	60	13	20	13	32	5
29	5	20	12	9	16	5	27	8	15	10	20	28	9	28	16	60	15
30	7	9	31	21	9	11	10	8	32	60	8	9	25	33	14	11	5
31	6	5	11	52	12	32	5	7	8	9	9	9	40	20	6	6	12
32	1	6	20	5	26	18	30	15	33	7	21	10	12	14	6	60	9
33	6	17	60	5	5	51	9	13	30	13	21	20	5	11	5	25	7
34	2	60	10	5	13	16	33	19	35	11	8	15	20	9	12	7	5

Table 1. Description of the dataset: the 34 studied cases

.

case	1 operator			2 operators			3 operators			4 operators			5 operators			6 operators			7 operators		
n°	**L1**	**U1**	**%**	**L2**	**U2**	**%**	**L3**	**U3**	**%**	**L4**	**U4**	**%**	**L5**	**U5**	**%**	**L6**	**U6**	**%**	**L7**	**U7**	**%**
1	303	303	0,00%	153	152	0,65%	108	103	4,63%	80	77	3,75%	75	64	**14,67%**	65	60	7,69%	60	60	0,00%
2	303	303	0,00%	166	152	8,43%	105	102	2,86%	96	79	**17,71%**	79	62	**21,52%**	61	60	1,64%	60	60	0,00%
3	303	303	0,00%	155	152	1,94%	104	102	1,92%	85	78	8,24%	70	65	7,14%	70	60	**14,29%**	60	60	0,00%
4	303	303	0,00%	161	152	5,59%	108	102	5,56%	83	78	6,02%	63	62	1,59%	60	60	0,00%	60	60	0,00%
5	303	303	0,00%	153	152	0,65%	118	103	**12,71%**	78	77	1,28%	76	63	**17,11%**	60	60	0,00%	60	60	0,00%
6	303	303	0,00%	153	152	0,65%	112	103	8,04%	95	80	**15,79%**	79	62	**21,52%**	60	60	0,00%	60	60	0,00%
7	303	303	0,00%	160	152	5,00%	118	104	**11,86%**	86	76	**11,63%**	72	62	**13,89%**	62	60	3,23%	60	60	0,00%
8	303	303	0,00%	173	152	**12,14%**	107	102	4,67%	88	78	**11,36%**	78	64	**17,95%**	63	60	4,76%	60	60	0,00%
9	303	303	0,00%	155	152	1,94%	117	104	**11,11%**	87	79	9,20%	77	63	**18,18%**	63	60	4,76%	60	60	0,00%
10	303	303	0,00%	164	152	7,32%	118	104	**11,86%**	84	77	8,33%	71	64	9,86%	60	60	0,00%	60	60	0,00%
11	303	303	0,00%	164	153	6,71%	107	103	3,74%	82	78	4,88%	75	63	**16,00%**	60	60	0,00%	60	60	0,00%
12	303	303	0,00%	162	152	6,17%	117	103	**11,97%**	84	77	8,33%	65	63	3,08%	60	60	0,00%	60	60	0,00%
13	303	303	0,00%	169	155	8,28%	116	105	9,48%	86	80	6,98%	65	63	3,08%	63	60	4,76%	60	60	0,00%
14	303	303	0,00%	154	152	1,30%	112	103	8,04%	81	79	2,47%	67	63	5,97%	60	60	0,00%	60	60	0,00%
15	303	303	0,00%	154	154	0,00%	104	104	0,00%	96	79	**17,71%**	76	63	**17,11%**	60	60	0,00%	60	60	0,00%
16	303	303	0,00%	163	152	6,75%	114	104	8,77%	85	78	8,24%	73	62	**15,07%**	60	60	0,00%	60	60	0,00%
17	303	303	0,00%	158	153	3,16%	123	103	**16,26%**	85	80	5,88%	83	63	**24,10%**	63	60	4,76%	60	60	0,00%
18	303	303	0,00%	161	152	5,59%	110	102	7,27%	91	79	**13,19%**	67	62	7,46%	60	60	0,00%	60	60	0,00%
19	303	303	0,00%	155	153	1,29%	105	103	1,90%	90	79	**12,22%**	74	62	**16,22%**	60	60	0,00%	60	60	0,00%
20	303	303	0,00%	152	152	0,00%	117	103	**11,97%**	84	79	5,95%	83	62	**25,30%**	60	60	0,00%	60	60	0,00%
21	303	303	0,00%	166	152	8,43%	110	104	5,45%	85	77	9,41%	74	65	**12,16%**	67	60	**10,45%**	60	60	0,00%
22	303	303	0,00%	155	152	1,94%	123	103	**16,26%**	90	79	**12,22%**	67	63	5,97%	63	60	4,76%	60	60	0,00%
23	303	303	0,00%	153	153	0,00%	121	102	**15,70%**	81	77	4,94%	70	61	**12,86%**	61	60	1,64%	60	60	0,00%
24	303	303	0,00%	159	153	3,77%	109	102	6,42%	85	78	8,24%	74	65	**12,16%**	65	60	7,69%	60	60	0,00%
25	303	303	0,00%	153	153	0,00%	120	102	**15,00%**	90	79	**12,22%**	67	63	5,97%	60	60	0,00%	60	60	0,00%
26	303	303	0,00%	173	153	**11,56%**	112	102	8,93%	87	77	**11,49%**	72	63	**12,50%**	60	60	0,00%	60	60	0,00%
27	303	303	0,00%	175	152	**13,14%**	104	104	0,00%	89	78	**12,36%**	70	62	**11,43%**	68	60	**11,76%**	60	60	0,00%
28	303	303	0,00%	159	153	3,77%	117	102	**12,82%**	83	77	7,23%	73	62	**15,07%**	62	60	3,23%	60	60	0,00%
29	303	303	0,00%	156	153	1,92%	110	102	7,27%	81	77	4,94%	75	62	**17,33%**	62	60	3,23%	60	60	0,00%
30	303	303	0,00%	165	152	7,88%	105	103	1,90%	88	77	**12,50%**	70	63	**10,00%**	63	60	4,76%	60	60	0,00%
31	303	303	0,00%	163	152	6,75%	116	102	**12,07%**	**84**	76	9,52%	67	64	4,48%	64	60	6,25%	60	60	0,00%
32	303	303	0,00%	164	153	6,71%	111	103	7,21%	86	78	9,30%	69	66	4,35%	68	60	**11,76%**	61	60	1,64%
33	303	303	0,00%	153	152	0,65%	107	102	4,67%	83	77	7,23%	73	62	**15,07%**	61	60	1,64%	60	60	0,00%
34	303	303	0,00%	162	153	5,56%	113	103	8,85%	85	78	8,24%	73	64	**12,33%**	68	60	**11,76%**	60	60	0,00%

Table 2. Summary of the experimentation results (in bold: improvements greater than 10%)

Table 3. Average and standard deviation of cycle durations of the different configurations, for Lx & Ux ($x \in [2, 6]$)

	2 operators			3 operators			4 operators			5 operators			6 operators		
	L2	***U2***	***%***	***L3***	***U3***	***%***	***L4***	***U4***	***%***	***L5***	***U5***	***%***	***L6***	***U6***	***%***
average	160	153	5%	112	103	8%	86.1	78.0	9%	72.4	63	13%	62.7	60	7,7%
standard deviation	6.44	0.71		5.77	0.84		4.31	1.10		4.92	1.13		2.90	0	

Instance	*number of tasks*	*Task duration max*	*Average duration*	*Task duration min*	*∑tasks durations*	*Standard deviation*	*Lin/complex*	***obtained sequence***
Bowman8	*8*	*17*	*9,38*	*3*	*75*	*4,31*	*C*	*1,2,4,3,6,5,8,7*
Jackson	*11*	*7*	*4,18*	*1*	*46*	*1,94*	*C*	*1,2,3,4,5,7,6,8,9,10,11*
Jaeschke	*9*	*6*	*4,11*	*1*	*37*	*1,45*	*C*	*1,2,3,4,5,6,7,8,9*
Mansoor	*11*	*45*	*16,8*	*2*	*185*	*14,8*	*C*	*3,1,2,4,5,6,7,8,9,10,11*

Table 4. Description of small-sized instances

Instance	*x*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Bowman8	Lx	75	42	28	22	20	17		
	Ux	75	38	26	21	18	17		
	%	0%	**10%**	7%	5%	10%	0%		
Jackson	Lx	46	24	19	13	12	11	9	7
	Ux	46	24	13	12	10	9	8	7
	%	0%	0%	**32%**	8%	**17%**	**18%**	**11%**	0%
Jaeschke	Lx	37	20	14	10				
	Ux	37	19	13	10				
	%	0%	5%	7%	0%				
Mansoor	Lx	185	98	68	50	45			
	Ux	185	93	65	49	45			
	%	0%	5%	4%	2%	0%			

Table 5. Results for small-sized instances

Instance	*number of tasks*	*Task duration max*	*Average duration*	*Task durationmin*	*∑tasks durations*	*Standard deviation*	*Lin/complex*	obtained sequence
Buxey	29	25	11,17	1	324	6,04	C	1,2,7,3,6,9,12,25,26,4,27,10,5,14,15,8,13,19,11,16,21,17,18,22,20,23,24,28,29
Gunther	35	40	13,8	1	483	12,09	C	17,1,2,5,10,12,3,6,4,7,8,9,11,14,18,13,15,19,16,20,21,22,25,30,23,26,31,24,32,27,33,28,34,29,35
Heskia	28	108	36,57	1	1024	33,09	C	2,1,3,4,5,6,8,17,19,21,22,23,24,26,7,9,20,25,27,10,18,11,12,13,15,14,16,28
Kilbrid	45	55	12,27	3	552	9,65	C	1,2,11,12,39,3,4,7,8,13,37,5,6,14,15,43,9,10,16,17,18,23,24,25,29,30,31,32,19,26,27,20,33,21,34,35,36,22,28,38,40,41,42,44,45
Lutz1	32	1400	441,9	100	14140	244,9	C	3,4,2,1,5,6,9,8,7,10,11,12,13,15,14,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32
Mitchell	21	13	5	1	105	2,97	C	1,2,3,4,21,5,6,7,8,14,9,10,11,12,13,15,16,18,19,17,20
Roszieg	25	13	5	1	125	2,81	C	1,2,3,4,5,8,6,9,10,7,11,12,13,15,17,14,16,19,20,23,18,21,22,24,25
Sauyer30	30	25	10,8	1	324	6,07	C	1,10,3,2,4,5,11,12,16,17,6,13,18,7,14,19,8,15,20,9,21,24,22,25,23,26,27,28,29,30

Table 6. Description of large-sized datasets

Instance	*x*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
Buxey	Lx	324	163	108	85	70	61	53	47	41	39	37	33	31	31	28	26	25
	Ux	324	162	108	81	66	55	47	41	39	34	31	30	27	25	25	25	25
	%	*0%*	*1%*	*0%*	*5%*	*6%*	***10%***	***11%***	***13%***	*5%*	***13%***	***16%***	*9%*	***13%***	***19%***	***11%***	*4%*	*0%*
Gunther	Lx	483	244	168	124	104	91	77	71	67	60	54	48	46	45	40		
	Ux	483	242	161	121	98	82	73	65	59	51	48	45	40	40	40		
	%	*0%*	*1%*	*4%*	*2%*	*6%*	***10%***	*5%*	*8%*	***12%***	***15%***	***11%***	*6%*	***13%***	***11%***	*0%*		
Heskia	Lx	1024	522	355	264	245	206	162	161	132	129	115	108					
	Ux	1024	514	344	258	209	175	157	132	129	111	108	108					
	%	*0%*	*2%*	*3%*	*2%*	***15%***	***15%***	*3%*	***18%***	*2%*	***14%***	*6%*	*0%*					
Kilbrid	Lx	552	278	190	156	116	97	88	79	70	61	57	56	56	55			
	Ux	552	277	184	139	111	93	80	71	63	56	55	55	55	55			
	%	*0%*	*0%*	*3%*	***11%***	*4%*	*4%*	*9%*	***10%***	***10%***	*8%*	*4%*	*2%*	*2%*	*0%*			
Lutz1	Lx	14140	7296	4920	3702	3100	2654	2284	1928	1898	1676	1628	1430	1400				
	Ux	14140	7078	4720	3598	2858	2392	2066	1858	1640	1514	1400	1400	1400				
	%	*0%*	*3%*	*4%*	*3%*	*8%*	***10%***	***10%***	*4%*	***14%***	***10%***	***14%***	*2%*					
Mitchell	Lx	105	56	37	28	23	21	18	16	16	15	13						
	Ux	105	53	35	27	22	18	16	15	13	13	13						
	%	*0%*	*5%*	*5%*	*4%*	*4%*	***14%***	***11%***	*6%*	***19%***	***13%***	*0%*						
Roszieg	Lx	125	64	43	34	29	23	21	18	16								
	Ux	125	63	42	32	26	21	19	17	16								
	%	*0%*	*2%*	*2%*	*6%*	***10%***	*9%*	***10%***	*6%*	*0%*								
Sauyer30	Lx	324	166	111	87	73	57	53	48	44	38	35	33	31	28	26	26	25
	Ux	324	162	109	82	66	55	48	42	38	35	31	29	27	25	25	25	25
	%	*0%*	*2%*	*2%*	*6%*	***10%***	*4%*	*9%*	***13%***	***14%***	*8%*	***11%***	***12%***	***13%***	***11%***	*4%*	*4%*	*0%*

Table 7. Results for large-sized datasets.